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MANAGING MARKET SHARE WHEN CONSUMERS SEEK VARIETY*

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Managing Market Share When Consumers Seek Variety

Abstract

We consider the managerial implications of a first-order Markov model of variety-seeking behavior. Construing the resulting equilibrium probabilities as expected choice shares, we examine the stability of these shares relative to changes in variety-seeking intensity, brand preferences, and perceptions of shared features. Complete model solutions are presented for two- and three-brand cases; in the n-brand case, approximations are analyzed. We find, regardless of the number of brands considered, that smaller preference brands tend to benefit most from variety-seeking and should try to appear unique; conversely, dominant brands benefit by subsuming the unique features of their smaller competitors. These theoretical results are illustrated by an empirical example, analyzing the soft-drink consumption histories of variety-seeking consumers.

Introduction

A number of researchers (Jeuland 1978, McAlister 1982, Lattin 1983) have developed models of consumer variety seeking behavior encoding the effects of entire purchase histories on subsequent selections within the same product class. The encoding mechanisms employed in such models often render them analytically intractable and make parameter estimation difficult. Givon (1983) has simplified the model, considering the effects of only the most recent selection on the one which follows it, implying a first-order Markov structure. This model assumes that the probability of switching from one brand to any other is solely a function of brand preferences and of predilection for switching; therefore, this probability is independent of any features of the brand switched-to, so long as it is different from the one switched-from. In contrast, we consider a first-order Markov model, proposed by Lattin and McAlister (1985), which accounts for similarities and disparities among different brands within the same product class. Specifically, the model suggests that someone prone to variety-seeking is more likely to choose a brand dissimilar to her last choice than she is to choose a similar one.

In formalizing the model, to the end of studying competitive interrelationships, Lattin and McAlister (1985) focussed on three key parameters: variety seeking intensity, brand preferences and perceptions of features shared by brands; while they deliberately held these parameters constant, we vary them, and, interpreting equilibrium probabilities as expected choice shares, investigate the likely change in choice shares brought about by such variation. With this information, managers can plan intervention strategies for markets in which consumers seek variety.

In order to produce such tools for managers, we must calculate and interpret the derivatives of equilibrium shares with respect to each of the parameters of the Markov model. To avoid the complexities this task eventually presents in the most general n-brand case, we begin by considering the two- and three-brand cases. The intuition developed in these simpler structures is shown to be consistent with the more general case.

Were consumers homogeneous with respect to variety seeking intensity, brand preferences and perceptions of features shared by brands, we could make simple predictions about market response based on the two-, three- and n-brand analyses. However, consumers are heterogeneous. In order to predict aggregated response, we must estimate the response of each consumer in a sample and aggregate these individual responses. In an illustrative empirical example, it is shown how these procedures can be applied and how such analyses can deepen insights into competitive interrelationships, providing guidelines for managerial action.

In the following section, we review the ideas behind the model and the model itself. Next, thorough analyses of two- and three-brand cases are presented. We then discuss the solution of the general n-brand model and present an empirical example applying the model to a collection of histories of variety-seeking consumers' choices among soft drinks. We end by summarizing the results of this work.

Modeling Framework

The model used to describe individual variety-seeking behavior (as described in Lattin and McAlister 1985) has four major assumptions.

1. A brand consists of a bundle of want-satisfying features. An individual's unconditional preference for a brand is given by the sum of the values of the brand's constituent features.

2. An individual's preferences for the available brands are influenced by that individual's consumption history. In the interest of parsimony, only the most recent previous choice is assumed to have any influence on individual preference.

3. The consumption of a particular feature will depress the value of that feature on the subsequent choice occasion. The degree of devaluation depends upon the individual's desire for variety. High intensity variety-seekers will almost completely discount recently consumed features. Low intensity variety-seekers will discount recently consumed features very little.

4. The conditional probability of brand choice by an individual is proportional to the individual's current preference for that brand (see Luce 1959).

Simply stated, the model holds that selection on one choice occasion affects preferences on the next choice occasion. Having just consumed a particular "bundle of features" (i.e., a brand), a variety seeker is less attracted to those features when making her next choice; therefore, brands composed of different features become relatively more attractive.

Describing the model analytically, let:

E = the choice set of brands for a consumer, defined as all brands chosen by that consumer over the time of the study

B_j = the j^{th} brand

π_j = the consumer's unconditional preference for B_j. This preference presents the total value to this consumer of all features - unique and shared - provided by B_j. Without loss of generality, these values are scaled so that

$$\sum_{B_j \in E} \pi_j = 1.$$

By definition $\pi_j = 0$ for all $B_j \notin E$ and $\pi_j = U_j - \sum_{\substack{B_k \in E \\ k \neq j}} S_{jk}$

U_j = the value to the consumer of B_j's unique feature(s)

S_{ij} = the value to the consumer of all want-satisfying features shared by brands B_i and B_j. By definition, $S_{ij} = S_{ji}$, and $S_{jj} = \pi_j$

V = a discount factor indicating the variety-seeking intensity of the consumer. $0 \leq V \leq 1$. If the consumer devalues recently consumed features completely, indicative of a high desire for variety, then $V = 1$

$\rho_{i|j} = \pi_j - V\, S_{ij}$ = the consumers conditional preference for B_i following the consumption of B_j. By definition $\rho_{j|j}=(1-V)\pi_j \leq \pi_j$.

$$a_{ji} = \frac{\rho_{i|j}}{\sum_{B_k \epsilon E} \rho_{k|j}} = \frac{\pi_i - V\, S_{ij}}{1 - V \sum_{B_k \epsilon E} S_{kj}} = \frac{\pi_i - VS_{ij}}{1 - VS_{.j}} \qquad [1]$$

= the consumer's conditional probability of choosing B_i following the consumption of B_j.

It is important to realize the distinction between π_i, the unconditional preference for B_i, and x_i, the steady-state choice share for B_i implied by the first order Markov choice process. The unconditional preference, π_i, is identical to the probability of choosing B_i if no previous choice had any impact on the current selection: (i.e in the absence of any variety-seeking behavior, implying V = 0). The steady-state choice share, x_i, reflects the impact of past choices and incorporates the influence of individual variety-seeking behavior on anticipated choice share.

Market Share as Equilibrium Probability

Having specified the transition probabilities, we can now solve for equilibrium choice shares. Although the solution will be presented for choice sets with arbitrarily numerous brands, the linear algebra required obscures managerial implications. We therefore begin by considering smaller choice sets, paying particular attention to stability and direction of change of equilibrium choice shares under changes in the model parameters. Specifically, we examine the consequences of: 1) changing the values of a brand's unique features; 2) changing the value of a brand's shared features; and 3) changing the intensity

of variety seeking behavior. Finally, we concern ourselves with inference in the complex n-brand case.

Two Brand Case

It is often the case, as in oligopoly, that there are few truly substantial brands in a market; in such situations, simple approximations based on insights about two-brand and three-brand cases often alleviate the complexity inherent in solving larger models, while affording the opportunity for a more rigorous mathematical analysis. In this section, we examine the implications of variety-seeking between two brands. While the structure of such a model appears deceptively simple, much of the strategy implied for choice sets of arbitrary size is foreshadowed in the treatment presented here.

Given two brands, B_1 and B_2, are the only relevant brands, we define:

$$a_{11} = \text{Prob (repeat purchase } B_1)$$

$$a_{22} = \text{Prob (repeat purchase } B_2)$$

Using these paramaters, we have two equations in the unknowns x_1 and x_2, creating a linear system of the form $X = XA$, where X is the row vector $[x_1\ x_2]$, the equilibrium choice shares for B_1 and B_2. The first order Markov transition matrix, A, is given by:

$$\begin{matrix} a_{11} & 1-a_{11} \\ 1-a_{22} & a_{22} \end{matrix}$$

The expected share, or equilibrium probability, for B_1 is:

$$x_1 = (1-a_{11})/[(1-a_{11})+(1-a_{22})]$$

The probabilistic definition of our model parameters imply:

$$a_{11} = \pi_1(1-V)/(1-VS_{.1})$$

$$a_{22} = \pi_2(1-V)/(1-VS_{.2})$$

where

$$S_{.1} = \pi_1 + S_{12} \qquad S_{.2} = \pi_2 + S_{12}$$

Substitution of these values yields:

$$x_1 = \frac{(\pi_1 - VS_{12})(1-VS_{.1})}{(\pi_1-VS_{12})(1-VS_{.1}) - (\pi_2-VS_{12})(1-VS_{.2})} \qquad [2]$$

Changing π's While Holding S_{12} Constant. Suppose a specific brand, say B_1, were to suddenly acquire, or discover, a unique feature previously overlooked; in a sense, this is the goal of a great deal of product awareness advertising. If this is the case, then our model well predicts the intuitive solution: adding unique features (without losing shared ones in the process) leads to an increased market share. To see this, we differentiate (2) with respect to U_1: (Recall that $\pi_1=U_1+S_{12}$, $\pi_2=1-U_1-S_{12}$.)

$$\frac{\partial x_1}{\partial U_1} = \frac{(1-V)(1-VS_{12})[(\pi_2-VS_{12})(1-VS_{.1})-(\pi_1-VS_{12})(1-VS_{.2})]}{(1-VS_{.1})^2(1-VS_{.2})^2\ [(\frac{\pi_1-VS_{12}}{1-VS_{.1}}) + (\frac{\pi_2-VS_{12}}{1-VS_{.2}})]^2} \qquad [3]$$

Both the numerator and denominator of this expression are positive for all values of $0<\pi_1<1$, $\pi_2=1-\pi_1$, $0\leq V<1$, $S_{12} \leq \min(\pi_1,\pi_2)$. This demonstrates that a sudden increase in unique features increases market share for the brand acquiring them. As we will see, this result will hold as well in situations with more brands.

Changing S_{12} While Holding π's Constant. As it will seldom be the case that one can ferret out an unending chain of new unique features through which to enhance choice share, it is useful to explore the effects of shifting preference from unique features to shared ones and vice-versa. Typical manifestations of the genre are found in a market leader's claiming market homogeneity in its advertising, or in an obscure brand's proclaimations of uniqueness. The question we eventually address is whether it is wise to shift perceptions of a brand to make it seem to share more of its valued features with some other brand: in the two brand case, we want to know which brand, the smaller or the larger, would benefit were they suddenly to be thought more alike.

Consider now $\partial x_1/\partial S_{12}$, the change in B_1's choice share brought about by shifting preference from unique to shared features while holding total brand preference constant. With $\pi_1=1-\pi_2$ constant, we compute the change in B_1's market share brought about by changing S_{12}. (Recall $\pi_1=U_1-S_{12}$ and $\pi_2=U_2-S_{12}$: hence decreasing S_{12} increases U_1 and U_2 and increasing S_{12} decreases U_1 and U_2.)

$$\frac{\partial x_1}{\partial S_{12}} = \frac{V(1-V)[(1-VS_{12})^2 - V\pi_1\pi_2](\pi_1-\pi_2)}{\left[\left(\frac{\pi_1-VS_{12}}{1-VS_{.1}}\right)+\left(\frac{\pi_2-VS_{12}}{1-VS_{.2}}\right)\right]^2 (1-VS_{.1})^2(1-VS_{.2})^2} \qquad [4]$$

The denominator in this expression is always positive. For all V other than 0 or 1, V(1-V) is positive. $(1-VS_{12})^2 - V\pi_1\pi_2$ is also always positive.[1] The sign of this expression is dictated by the sign of $(\pi_1-\pi_2)$. If $\pi_1>\pi_2$, then $\partial x_1/\partial S_{12} < 0$.

The negativity of $\partial x_1/\partial S_{12}$ implies that, in a choice set with two brands, the larger preference brand gains share if the brands can be made to seem to derive more of their value from shared features; the smaller preference brand gains share if the two brands can be made to seem to derive less.

Changing V. Perhaps the parameter most germane to the entire model is V, the variety seeking parameter. Through investigating the implications of changing V, we essentially ask whether a brand ought to try to perpetuate and reward people's desire for variety.

The change in B_1's choice share brought about by changing V is

$$\frac{\partial x_1}{\partial V} = \frac{(\pi_1 - \pi_2)(VS_{12}^3 - 2VS_{12}^2 + (1-V^2\pi_1\pi_2)S_{12} - \pi_1\pi_2)}{\left[\frac{\pi_1 - VS_{12}}{1-VS_{.2}} - \frac{\pi_2 - VS_{12}}{1-VS_{.1}}\right]^2 (1-VS_{.1})^2(1-VS_{.2})^2} \qquad [5]$$

and $(\pi_1 - \pi_2)$ is positive so long as $\pi_1 > \pi_2$. The other term in the numerator is positive only for values of S_{12} very near $\min(\pi_1,\pi_2)$. From this we can see that for those situations in which the brands have very little in common (S_{12} is small), increasing V increases the choice share of the smaller preference brand.

At some point the brands are so similar that increases in V begin to increase the choice share of the larger preference brand.

Summary. For the case in which there are only two relevant brands, we find that a brand's choice share increases with increases in the value of the brand's unique features. The choice share of the larger preference brand of the two increases as the value of their shared features increases. Finally, choice share of the smaller preference of the two brands increases with V unless the smaller preference brand is "extremely similar" to the larger preference brand.

Three-Brand Case

In many ways, the three-brand case is closer to generality than it is to the two-brand case; while the two-brand case is severely limited by summation constraints on any pair of brands (being that there is only one pair), this is not so in the three-brand case. Further, since large choice sets can be logically partitioned into an "us-them-everyone else" kind of trichotomy, intuition garnered from the following analysis should be particularly useful.

Extending the conventions of the two brand case, we define for $j \neq i$: $1 \leq i, j \leq 3$; $0 < a_{ji} \leq 1$:

$$a_{ji} = P_{i|j} = P(\text{purchase } B_i \text{ after } B_j)$$

Since some B_i must be purchased after each B_j, we know $a_{j1}+a_{j2}+a_{j3}=1$ for any j; hence:

$$a_{11} = P(\text{repeat purchase } B_1) = 1-a_{12}-a_{13}$$

$$a_{22} = P(\text{repeat purchase } B_2) = 1-a_{21}-a_{23}$$

$$a_{33} = P(\text{repeat purchase } B_3) = 1-a_{31}-a_{32}$$

Using a procedure later applied to the n-brand case, we let $X = [x_1\ x_2\ x_3]$, the vector of equilibrium market shares, and set up the system $X = XA$: the first order Markov transition matrix, A, is given by

$$A = [a_{ji}] \qquad 1 \leq i,j \leq 3$$

$$[x_1\ x_2\ x_3] = [x_1\ x_2\ x_3] \begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

In the appendix, we report the closed form expressions for x_1, x_2, and x_3 as functions of the transition probabiltities. We avoid rewriting x_1, x_2 and x_3 as functions of V, π's and S_{ij}'s because the complex analytical expressions offer little insight. The expressions for $\partial x_j/\partial \pi_j$, $\partial x_j/\partial S_{ij}$ and $\partial x_j/\partial V$ are similarly obscure. We opt, instead, to explore the ways in which choice shares change through simulation.

Initially, we consider the case in which none of the brands share any features (Figure 1A). Consistent with the finding that $\partial x_j/\partial U_j>0$ in the two brand case, we find that in this three brand case, larger preference brands have

FIGURE 1
Illustration of the Three-Brand Case with
$\pi_1 = .59$, $\pi_2 = .27$, $\pi_3 = .14$, $V = .8$

Fig. 1a: $S_{12} = S_{13} = S_{23} = 0$;
$x_1 = .47$, $x_2 = .34$, $x_3 = .19$

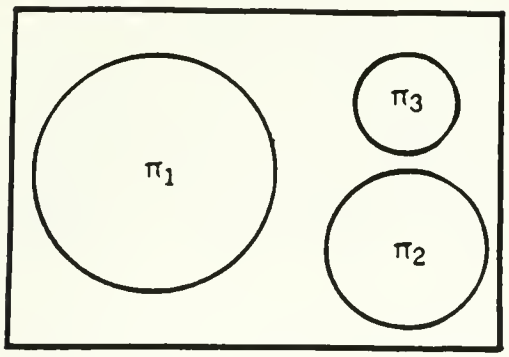


Fig. 1b: $S_{13} = .07$, $S_{12} = S_{23} = 0$;
$x_1 = .48$, $x_2 = .37$, $x_3 = .15$

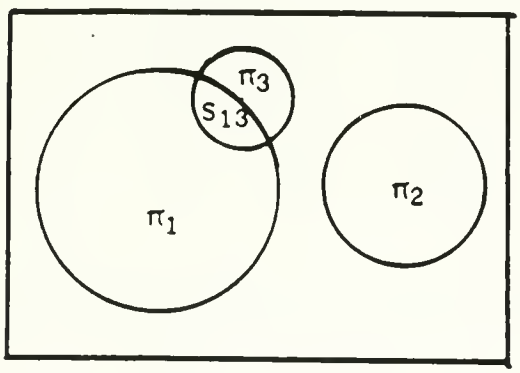


Fig. 1c: $S_{13} = \pi_3$, $S_{12} = S_{23} = 0$;
$x_1 = .49$, $x_2 = .40$, $x_3 = .11$

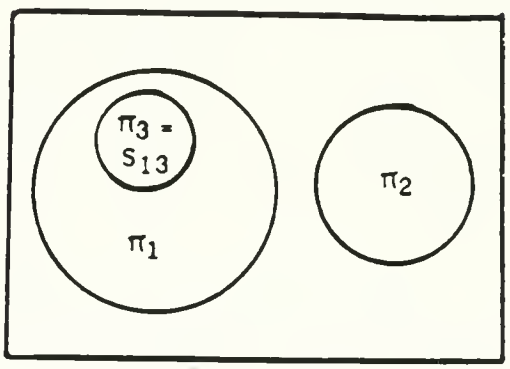
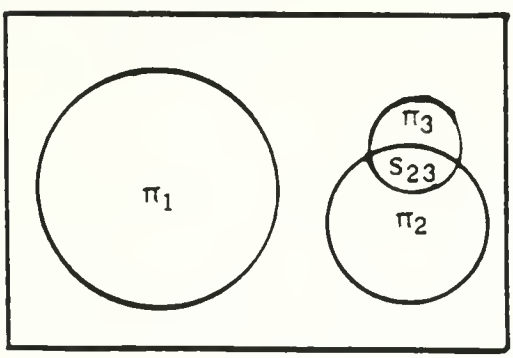


Fig. 1d: $S_{23} = .07$, $S_{12} = S_{13} = 0$;
$x_1 = .50$, $x_2 = .33$, $x_3 = .17$



larger choice shares ($\pi_1>\pi_2>\pi_3 \rightarrow x_1>x_2>x_3$). Also consistent with the two brand case results, we find that smaller preference brands gain more from consumer variety seeking ($\pi_1>\pi_2>\pi_3 \rightarrow x_3-\pi_3 > x_2-\pi_2 > x_1-\pi_1$). In the case depicted by Figure 1A, the two smaller preference brands gain ever more share and the larger preference brand loses ever more share as variety seeking intensifies $\partial(x_3-\pi_3)/\partial V > 0$, $\partial(x_2-\pi_2)/\partial V > 0$, $\partial(x_1-\pi_1)/\partial V < 0$).

[Figure 1 Here]

The effects of shared features on equilibrium choice shares do not generalize so simply from the two- to the three-brand case. In the two-brand case, the larger preference brand benefited from sharing features, the smaller preference brand, from seeming unique. In the three brand case, a brand benefits most by remaining unique while the other two brands are made to seem to share more features. If a brand must share features, then it is hurt least by sharing features with the smaller preference of the other two brands. To illustrate these two points, consider Figures 1B and 1C.

These figures represent a three brand choice set in which preferences are distributed as in Figure 1A, but perceptions of shared features differ. In Figure 1B, B_3 derives some of its value from features shared with B_1. In figure 1C, B_3 derives all of its value from features shared with B_1. B_2's gain due to variety seeking ($x_2-\pi_2$) grows as perceptions of shared features changes from that depicted by Figure 1A to that depicted by Figure 1B to that depicted by Figure 1C. B_3 would experience a similar gain if it were left unique and B_2 began to share features with B_1. B_1, which loses share due to variety seeking, would reduce its loss if left unique while B_2 and B_3 began to share features.

Hence we see that any brand benefits when the other two brands are made to share more features.

The magnitude of the loss that a brand experiences as it begins to share more features is a function of the preference for the brand with which it shares features. Figure 1D represents a situation in which preferences are distributed as before but perceptions of shared features differ. In Figure 1A, B_3 shares no features. In Figure 1B, B_3 shares some features with B_1. In Figure 1D, B_3 shares some feature with B_2. B_3's gain in share due to variety seeking ($x_3-\pi_3$) is greatest when it is unique (Figure 1A). If, however, B_3 must share features with either B_2 (Figure 1D) or B_1 (Figure 1B), it retains more of its choice share advantage by sharing with B_2, the smaller preference of the two.

These results reinforce and extend insights gleaned from the two-brand case. In addition to knowing that a brand's share is increased by adding unique features and that smaller preference brands benefit more from consumer variety seeking, we now see that any brand can gain share by making other brands seem to share features and that it is less detrimental to share features with smaller rather than larger preference competitors.

N-Brand Case

In the most general case, we consider variety-seeking among n brands, interpreting several key derivatives whose properties have been catalogued for the two- and three-brand cases: $\partial[X]/\partial V$, $\partial[X]/\partial U_j$, $\partial[X]/\partial S_{12}$ (holding all π_j

constant). Although it is possible to obtain precise solutions, in linear algebraic terms, for each derivative, these have been relegated to the appendix. We have instead chosen to scrutinize approximations to these derivatives, as the functional forms of the approximations help provide intuition; empirical evidence shows the approximations to be quite good.[2] All theoretical results quoted in this section are demonstrated in the appendix; we have therefore referenced specific equations relative to the numbering scheme employed there.

Solving for $\{x_j\}$, the Equilibrium Market Shares. The nxn first-order transition matrix [A] has elements a_{ji} = p [chooses B_i given that B_j was chosen last time]. As before, $a_{ji} = (\pi_i - VS_{ij})/(1-VS_{\cdot j})$. Equilibrium probabilities, and hence expected choice shares, are given by the 1xn vector [X] where

$$[X] = [X]\,[A] \qquad [I1]$$

The solution is:

$$x_j = |\det(j^{th} \text{ minor of last column } A^*)/\det(A^*)| \qquad [I4]$$

where:

A^* = nxn matrix with elements:

$$a^*_{ji} = \begin{cases} 1 & \text{if } i=n \\ a_{ji} & \text{if } 1 \le i < n,\ j \ne i \\ a_{ji}-1 & \text{if } 1 \le i < n,\ j = i \end{cases}$$

Changing V. Aside from the formal solution presented by [I9], we can approximate the derivatives of choice shares with respect to V:

$$\frac{\partial}{\partial V}[X] \approx [X]\left(\frac{\partial}{\partial V}[A]\right) \qquad [I13]$$

$$\rightarrow \frac{\partial}{\partial V}(x_i) \approx \sum_{j=1}^{n} x_j\,(\pi_i S_{.j} - S_{ij})/(1-VS_{.j})^2 \qquad [I14]$$

The sign of $\partial x_i/\partial V$ is not obvious for [I14], so we consider the special case in which $S_{ij} = 0$ for all $i \neq j$. Letting B_1 and B_2 be two brands with $\pi_1 > \pi_2$, we have

$$\partial(x_1-x_2)/\partial V = (\pi_1-\pi_2)\left[\sum_{j>2}(x_j\pi_j/(1-V\pi_j)^2)\right] + \\ - (1-(\pi_1-\pi_2))\left[\sum_{j=1}^{2}(x_j\pi_j/(1-V\pi_j)^2)\right] \qquad [6]$$

Notice first that if there were no brands in the choice set other than B_1 and B_2, the first term in [6] would go to zero and the sign of [6] would be negative, reinforcing our intuition that smaller preference brands gain more from variety seeking than do larger preference brands. Notice also that if there are many brands in the choice set, but if B_1 is a dominant brand (π_1 is very large), then $(x_1\pi_1/(1-V\pi_1)^2$ will be large and all other $(x_j\pi_j/(1-V\pi_j)^2$ will be small, resulting in a negative value for [6].[3]

In general, the determination of which brands will benefit from variety seeking and which will be hurt requires consideration of the distribution of brand preferences and the pattern of shared features.

Changing U_k. Because the π's are constrained to sum to one, we must specify the way in whch we expect all π_j ($j \neq k$) to change if we change the value of U_k, and hence π_k. In the two-brand case, we opted for simplicity and assumed that a change in the value of one brand's unique feature was exactly offset by a change in the value of the other brand's unique feature. The value of their shared feature was assumed constant. In the n-brand case we consider a more complex rule for allocating the change; each feature other than U_k is assumed to absorb the change in proportion to its value (i.e. $\partial U_i/\partial U_k = -U_i/1-U_k$ for $i \neq k$, and $\partial S_{ij}/\partial U_k = -S_{ij}/1-U_k$). This is actually the logical sharing rule, given the feature-based orientation of the model; using the scaling method of equation [1], the Luce choice axiom automatically accounts for the changes in each feature parameter (U's and S_{ij}'s) when a new unique feature is added.

We fix a brand, B_1, and analyze approximations for $\partial x_i/\partial U_1$, the change in B_i's market share dictated by B_1's gaining unique features. Since there is no preferred scheme for numbering the brands, no generality is lost by fixing our attention on B_1:

$$\partial x_i/\partial U_1 \approx \sum_{j=1}^{n} (\partial a_{ji}/\partial U_1)(x_j) \qquad [I16]$$

where the transition probability derivatives are given by:

$$\partial a_{ji}/\partial U_1 = \begin{cases} -a_{ji}/(1-U_1)(1-VS_{.j}) & \text{for } i \neq 1,\ j \neq 1 \\ (1-a_{ji})/(1-U_1)(1-VS_{.j}) & \text{for } i=1,\ j \neq 1 \\ -a_{ji}(1-V)/(1-U_1)(1-VS_{.j}) & \text{for } i \neq 1,\ j=1 \\ (1-a_{ji})(1-V)/(1-U_1)(1-VS_{.j}) & \text{for } i=1,\ j=1 \end{cases} \qquad [I17]$$

Looking over the expressions for $\partial a_{ji}/\partial U_1$, it is clear that for brands other than B_1, all transition probability derivatives are negative, so the other

brands lose share; conversely, for B_1, gaining unique features guarantees an increase in market share. We also note that, on the average, a_{ji} (and therefore, by the previous formulas, $\partial a_{ji}/\partial U_1$ for $i \neq 1$) is roughly proportional to π_i, so a change in U_1 allows B_1 to exchange market share with its competitors relative to the size of their shares. Due to the term $(1-U_1)$ in the denominator of each transition coefficient derivative, we see that the effect is magnified by the level of B_1's uniqueness before it obtained its new unique feature; this demonstrates that increments in uniqueness become ever more beneficial. As long as $(1-VS_{.j})$ doesn't vary wildly across the brands (and this is quite likely in large choice sets), we see that the worth of new unique features is dictated completely by relative brand preference and by the level of uniqueness of the brand gaining it.

Changing S_{12} with π's Held Constant. We now consider the impact on choice shares of making two brands appear more similar without changing total preference for any brand. We fix two brands, B_1 and B_2, and calculate $\partial x_K/\partial S_{12}$ holding all π's constant:

$$\partial x_1/\partial S_{12} \approx [Va_{11}/(1-VS_{.1})](x_1) - [V(a_{21}-1)/(1-VS_{.2})](x_2) \qquad [I19]$$

$$\partial x_2/\partial S_{12} \approx [V(a_{12}-1)/(1-VS_{.1})](x_1) + [Va_{22}/(1-VS_{.2})](x_2)$$

$$\partial x_K/\partial S_{12} \approx [Va_{1k}/(1-VS_{.1})](x_1) - [Va_{2k}/(1-VS_{.2})](x_2) \quad \text{for } k>2$$

That the approximation for $\partial x_K/\partial S_{12}$ for $k>2$ is positive demonstrates that the competition can expect to benefit when two brands trade their unique features for shared ones. It is for this reason that a product line should aim for diversity to benefit most from variety-seeking. As for B_1 and B_2, examination

of the expessions above yields two conclusions: 1) the more numerous the brands, the more likely it is that any two will both lose share by appearing more alike; and 2) if either is to benefit from such a move, it is the larger of them, particularly when variety-seeking is intense.

An Empirical Example

Thusfar, our analyses have implicitly assumed homogeneous preferences, perceptions, and variety seeking intensities for all consumers in the market. Such assumptions allowed us to develop insight concerning the forces that affect share when consumers seek variety. To engineer a managerial tool based on these theoretical relationships, we must recognize consumer heterogeneity and incorporate it into a plan for data analysis. In this section we present such a tool, illustrating its usefulness with data from a small sample of variety seeking consumers identified by Lattin and McAlister (1985).

Starting with 27 consumers' histories of soft drink consumption, Lattin and McAlister (1985) estimated a set of π's, S_{ij}'s and a V for each using equation [1] and a constrained non-linear program. Through bootstrapping, they identified nine of those consumers as variety seekers (V's statistically distinguishable from 0). Lattin and McAlister (1985) then examined competitive interrelationships among brands given that variety seeking intensities, brand preferences and perceptions of shared features remained constant. We use their data to examine competitive interrelationships when variety seeking intensity, brand preferences or perceptions of shared features change.

To estimate this variety seeking segment's response to changes in the parameters of this process, individual level derivatives are calculated using equations [I5], [I12] and [I15] and then averaged, weighting each consumer's derivative by her relative frequency of consumption.[4]

Across the nine variety-seekers, eight different soft-drinks were consumed: Coke, Dr. Pepper, Pepsi, 7-Up, Fruit Flavor (aggregated Sprite, Grape, Orange), Tab, Cola (Shasta), and Diet Fruit Flavor (aggregated Diet 7-Up, Diet Lemon-Lime). The observed segment shares for the eight soft drinks are given in Table 1; notice that the observed shares closely match those predicted by the model (equation [I3]).

Table 1
Observed and Estimated Segment Shares for Eight Brands
Chosen by Nine Variety-Seeking Consumers

B_i	n_i/N = observed segment share*	x_i = estimated segment share
Coke	36.6%	34.4%
Dr. Pepper	0.8	0.9
Pepsi	23.5	24.5
7-Up	19.3	18.4
Fruit Flavor	1.9	4.6
Cola	5.4	4.8
Tab	2.1	2.1
Diet Fruit	7.4	8.3

* n_i = the number of times B_i was chosen and
N = the number of times any brand was chosen

We estimate the segment's response for these eight brands. If any particular brand is not in an individual's choice set, all derivatives of that brand's share are zero with respect to any parameter in the model.

To determine which brands are hurt and which are helped by variety seeking, and how much share is entailed, we consider $\partial x_i/\partial V$ as calculated by [I5]:

Table 2

Change in Segment Share Brought About by Changing Variety Seeking Intensity ($\partial x_i/\partial V$) for a Sample of Nine Variety Seeking Consumers

Brand i	$\partial x_i/\partial V$	Average Choice Share for B_i for Those Consumers Who Ever Chose B_i	Number of Consumers Who Ever Chose B_i
Coke	-.214	.47	7
Dr. Pepper	.045	.06	1
Pepsi	.081	.28	6
7-Up	.016	.27	5
Fruit	.070	.12	3
Cola	-.032	.62	1
Tab	*	.25	1
Diet Fruit	.016	.28	3

* This effect is negligible ($-0.01 < \partial x_i/\partial V < 0.01$)

Reading down the second column of Table 2 we see that Coke and Cola expect to lose share if variety seeking intensifies while all other brands expect to gain share. Taking segment share as an indication of brand preference, it is not surprising that Coke (with a segment share of 36.6) should expect to lose share as variety seeking intensifies. We expect large preference brands to be hurt by such intensification. Taking Cola's sement share (5.4) as an indication of its preference, we find its negative derivative inconsistent with earlier intuition. To understand why Cola should expect to lose share as V intensifies, we need to investigate heterogeneity in the make-up of consumer's choice sets. The third column of Table 2 reports each brand's average choice share <u>among those consumers who ever chose the brand</u>. Notice that Cola had the highest

value and Coke the second highest value in this column. The effects hypothesized earlier occur at the individual level and are in fact consistent with Table 2. The single consumer who chooses Cola evidently chooses it frequently. If this consumer becomes a more intense variety seeker, Cola should be chosen less frequently. Hence, we can say that brands that are highly preferred by those consumers who ever choose them are the brands that should expect to lose share as variety seeking intensifies.

Much of this difficulty inference is largely an artifact of aggregation; what appears counterintuitive relative to aggregate market shares makes perfect sense when dissected at the disaggregate level. Although such epithets as 'smaller brands tend to benefit from variety seeking' are, in general, plausible, it is quite dangerous to infer that, for example, a 'small brand' is any brand with a small aggregate share. To highlight this distinction, we consider two brands with equal aggregate shares, both medium size for the market. Further, let us assume that the vast majority of consumers in our sample have chosen the first brand, but it is never a large preference brand for any of them; contrarily, the second brand is chosen by few consumers, but those who choose it tend to have very high preference for it. Through analysis at the disaggregate level, it should be clear that the first brand, with its small preference share, will benefit from variety seeking, while the second brand, with its large preference share, will be hurt by variety seeking. Although these effects carry over upon aggregation, the two brands could well have equal aggregate market shares, though individual choice shares are very different among those consumers who did purchase them.

Such problems notwithstanding, we can get some insight into the magnitude of aggregate response by considering the number of consumers who ever chose each brand. Coke's derivative is much larger in absolute value than is Cola's because Coke is chosen by seven of the nine consumers and Cola by only one of the nine. Therefore, Coke's derivative has only two zeros averaged in while Cola's derivative has eight.

Similarly, consider Pepsi, 7-Up, Tab and Diet Fruit. These brands have approximately the same average choice shares among consumers who ever choose them and the rank order of $\partial x_i/\partial V$ for these brands is consistent with the rank order of their subsegments. These effects are driven by the patterns of brand preference, perceptions of shared features and variety seeking intensity among the consumers who who choose the brands. Consideration of $\partial x_i/\partial U_j$ and $\partial x_i/\partial S_{jk}$ holding all π's constant give further insight into these relationships.

Table 3

Change in Segment Share Brought About by Changing The Value of Unique Features ($\partial x_i/\partial U_j$) for a Sample of Nine Variety Seeking Consumers:

B_j \ B_i:	Coke	Dr. Pepper	Pepsi	7-up	Fruit	Cola	Tab	Diet Fruit
Coke	2.06	-0.54	-1.10	-0.08	-0.79	-0.01	-0.01	-0.07
Dr. Pepper	-0.33	0.60	-0.09	**	**	**	**	**
Pepsi	-0.72	-0.05	2.69	-2.11	-0.08	-0.01	-0.02	-0.03
7-up	-0.12	**	-1.34	2.33	-0.13	**	-0.03	-0.06
Fruit	-0.70	**	-0.05	-0.04	1.03	**	**	-0.02
Cola	-0.06	**	-0.05	**	**	0.05	**	-0.04
Tab	-0.02	**	-0.02	-0.03	**	**	0.07	**
Diet Fruit	-0.08	**	-0.05	-0.06	-0.03	-0.03	**	0.24

** These effects are undefined because the corresponding row brand and column brand do not appear together in any of the nine consumer's choice sets.

Scanning down a column of Table 3 we get a sense of the vulnerability of the brand on the column head to all other brands in the sample. Coke, for instance, is about equally vulnerable to Pepsi and Fruit Flavor; should either Pepsi or Fruit manage to increase perceptions of uniqueness, Coke might expect to lose a substantial amount of share. Vulnerability is reciprocal, but not necessarily transitive. For example, Coke, as mentioned above, is vulnerable to Pepsi and Fruit Flavor. Pepsi is vulnerable to Coke, but not particularly vulnerable to Fruit Flavor. These non-transitivities occur because of the heterogeneity in the composition of these consumers' choice sets. Coke tends to occur with Pepsi in some choice sets and with Fruit Flavor in others. Pepsi, on the other hand, rarely occurs in choice sets with Fruit Flavor.

While Table 3 describes one brand's vulnerability to other brands' becoming more unique, Table 4 summarizes a target brand's vulnerability when other brands become less unique by sharing more features with the target. Reading down column 1 of Table 4, we see that Coke can expect to gain share if Dr. Pepper, Pepsi or Fruit Flavor can be made to share more of their valued features with Coke. Coke can expect to lose share if it begins to share more of its valued features with any of the other brands. This occurs because Coke is a fairly large preference brand in those choice sets in which it occurs with Dr. Pepper, Pepsi and Fruit Flavor. Coke is typically a small preference brand in those choice sets in which it occurs with other brands. Similarly, we see that Pepsi tends to be a dominant brand when it occurs with 7-Up, and 7-Up tends to be dominant when it occurs with Fruit Flavor. Consistent with our insights from Table 2, we see that Diet Cola is a dominant brand in the choice set of the one consumer who ever chooses it. Again, note that dominance os defined on a choice

set by choice set basis. Aggregate segment share is an unreliable surrogate for this measure.

Table 4

Change in B_i's Segment Share Brought About by Changing the Value of Shared Features ($\partial x_i / \partial S_{ij}$) while holding Total Preference for Brands Constant for a Sample of Nine Variety Seekers

B_i: / B_j	Coke	Dr. Pepper	Pepsi	7-up	Fruit	Cola	Tab	Diet Fruit
Coke	***	-0.52	-0.97	0.03	-0.60	-0.03	*	-0.02
Dr. Pepper	0.47	***	*	**	**	**	**	**
Pepsi	0.71	*	***	-1.97	-0.05	0.02	-0.01	*
7-up	-0.14	**	1.80	***	-0.18	**	-0.01	*
Fruit	0.54	**	-0.02	0.02	***	**	**	*
Cola	-0.08	**	-0.07	**	**	***	**	-0.05
Tab	-0.01	**	*	*	**	**	***	**
Diet Fruit	-0.02	**	-0.01	0.02	*	-0.02	**	***

* These effects are negligible ($-0.01 < \partial x_i \partial S_{ij} < 0.01$)

** These effects are undefined because the corresponding row brand and column brand do not appear together in any of the nine consumer's choice sets.

*** Diagonal entries are undefined since this table considers the impact on one brand of increasing the value of features shared woth some <u>other</u> brand. Diagonal entries would indicate the effect of a brand on itself.

Table 5 looks at this data in another way. Rather than considering the impact on a single brand of increasing the value of features it shares with other brands, we consider the impact on all brands of some particular pair of brands beginning to share more features.

Table 5

Change in B_i's Segment Share Brought About by Changing the Value of Shared Features ($\partial x_i/\partial S_{jk}$) while holding Total Preference for Brands Constant for a Sample of Nine Variety Seekers

B_i: / S_{jk}	Coke	Dr. Pepper	Pepsi	7-up	Fruit	Cola	Tab	Diet Fruit
$S_{Coke,Pepsi}$	0.71	0.08	-0.97	0.07	0.02	*	*	*
$S_{7\text{-}Up,Fruit}$	0.10	**	0.06	0.02	-0.18	**	**	*

* These effects are negligible ($-0.01 < \partial x_i/\partial S_{jk} < 0.01$)

** These effects are undefined because the corresponding row brand and column brand do not appear together in any of the nine consumer's choice sets.

As we knew from Table 4, increasing $S_{Coke,Pepsi}$ will help Coke and hurt Pepsi; increasing $S_{7\text{-}Up,Fruit}$ will help 7-Up and hurt Fruit. Table 5 reiterates this information and goes beyond Table 4, giving us a sense of how changes in S_{ij} will influence brands other than B_i and B_j. As predicted in the three- and n-brand cases, these other brands do benefit when the target pair begins to share more features.

Summary

Using Lattin and McAlister's (1985) estimates of π's, S_{ij}'s and V's for a sample of variety seeking consumers, we estimated consumer choice shares using equation [13]. A weighted average of these estimates proved to be a very accurate estimate of observed segment shares. We then calculated precise solutions for $\partial x_i/\partial V$, $\partial x_i/\partial U_k$ and $\partial x_i/\partial S_{jk}$ (holding π's constant) for these consumers, and took weighted averages to estimate segment response to changing

variety seeking intensity, brand preferences and perceptions of shared features. Examination of these segment response measures yields insight into competitive interrelationships among brands and confirms theoretical findings. Brands that are dominant in the choice sets in which they occur are penalized most by variety seeking. Any brand benefits by increasing the value of its unique features. Dominant brands benefit by increasing the value of features shared with small brands, while it is benefical for small brands to seem unique. Finally, any brand can benefit if two other brands can be made to seem to derive more of their value from shared features.

Conclusion

Perhaps conclusions are best drawn through retracing our steps; the natural evolution of managerial insight follows well from the two- through the three- to the n-brand case.

Despite the admitted limitations of the two-brand analysis, many of the most general and important results are completely spelled out there: smaller brands tend to benefit from variety seeking; smaller brands should attempt to channel shared aspects to unique ones; a new unique feature is beneficial to any brand. The two-brand case is limited by its construal of either brand as an island, incapable of affecting the other through interaction with hypothetical competitors. As such, two-brand analyses tend to churn out coarse, though valid, presriptions, ignoring the subtle interplay beyond the direct control of a target brand.

The three-brand case accounts for this interplay, allowing analysis of two brands' interactions and their effect on a third brand in the same choice set. We find that, in addition to the results from the two brand case, that a brand benefits most by making its competitors appear more similar, and if a brand must share features with another brand, that brand should be as small as possible.

The n-brand case helps demonstrate the previous findings in choice sets with numerous brands. When precise solutions become computationally unruly, approximations can accurately summarize how large choice sets respond to certain parametric fluctuations; moreover, the functional forms of the approximations often help identify pertinent forces driving the patterns of response.

Before any of the model's suggestions are followed too rigidly, we must remember that all estimation and analysis have been carried out at the disaggregate level. As discussed in the empirical example, interpreting disaggregate model parameters like 'choice share' as their similarly named aggregate counterparts assumes a strict homogeneity unlikely to exist in an actual sample. Although we have presented several techniques to counter these inferential difficulties, it is crucial to remember that heterogeneity is a foundational element in any market-level analysis.

To successfully manage market share, a manager must therefore be well versed not only in the likely effects of his actions relative to the individual variety seeker, but also in how individual variety seekers fit into the heterogeneous mosaic of the consumer pool in its entirety.

FOOTNOTES

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[1] To see this, realize that $(1-VS_{12})>(1-S_{12})>\pi_1>0$ and $(1-S_{12})>\pi_2>0$, therefore $(1-VS_{12})^2 > \pi_1\pi_2 > V\pi_1\pi_2$.

[2] For the soft drink data presented in the following section, regressions of the approximated derivatives on their exact counterparts produced adjusted R^2 values of 89.1% for $\partial[x]/\partial V$, and at least 99% for all derivatives of [X] with respect to the various U_j and Si_j parameters. Moreover, the constants in these regressions were never significant; the approximations consistently underestimated their corresponding exact values, by 25% for $\partial[X]/\partial V$ and by less than 10% for $\partial[X]/\partial U_j$ and $\partial[X]/\partial Si_j$.

[3] It is true that if π_1 is very large, then $(\pi_1-\pi_2)$, the multiplier of the positive term in [I15], will be large, while $(1-(\pi_1-\pi_2))$, the multiplier of the negative term will be small. However, as π_1 grows, $x_1\pi_1/(1-V\pi_1)^2$ becomes large faster than $(1-(\pi_1-\pi_2))$ becomes small.

[4] Letting n_c = the number of choices made by consumer c and letting N = the total number of shoices made by these nine variety seeking consumers, consumer c's relative frequency of consumption is given by n_c/N.

Appendix I

Solving For Choice Share in the Three-Brand Case

Recall that transition probabilities are given by:

$$a_{ji} = P[\text{purchase } B_j \text{ after } B_i] \quad \text{for } 0 \leq a_{ij} \leq 1 \; ; \quad i,j=1,2,3.$$

We therefore have the transition matrix:

a_{11}	a_{12}	a_{13}
a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}

The market share (equilibrium probabilities) for the three brands are:

$$x_1 = [a_{12}(a_{23} + (a_{13}a_{21})/(1-a_{11})) - a_{13}(1-a_{22} - (a_{12}a_{21})/(1-a_{11}))]/D$$

$$x_2 = (1-a_{11})[a_{23} + (a_{13}a_{21})/(1-a_{11})]/D$$

$$x_3 = (1-a_{11})[1-a_{22} - (a_{12}a_{21})/(1-a_{11})]/D$$

where

$$D = (1-a_{11}-a_{12})[a_{23} - (a_{13}a_{21})/(1-a_{11})] - (1-a_{11}-a_{13})[1-a_{22} - (a_{12}a_{21})/(1-a_{11})]$$

Model Solution

Given $A = [a_{ji}]$ $1 \leq i,j \leq n$ and $a_{ji} = P(B_j \text{ chosen after } B_i) = (\pi_i - VS_{ij})/(1-VS_{.j})$:

$$X = XA \quad \rightarrow \quad 0 = X\,[I-A] \qquad [I1]$$

Since $\sum_{i=1}^{n} a_{ji} = 1$, the singularity of [I-A] is expected. Replacing the last column of [I-A] by a 1xn vector of ones captures the constraint $\sum_{j=1}^{n} x_j = 1$ and yields a system equivalent to [I1]:

$$XA^* = e_n = (00\ldots01) \qquad [I2]$$

where $A^* = [a^*_{ji}]$ and

$$a^*_{ji} = \begin{cases} -a_{ji} & \text{for } i<n,\ j \neq i \\ 1-a_{ji} & \text{for } i<n,\ j=i \\ 1 & \text{for } i=n,\ \text{all } j \end{cases}$$

The solution to [I2] is given by:

$$[X] = e_n\,[A^*]^{-1} \qquad [I3]$$

$$x_k = |\text{Det } (k^{th} \text{ minor of last column of } A^*)/\text{Det } [A^*]| \qquad [I4]$$

Calculating Derivatives of Equilibrium Market Shares

To calculate the derivative of [X] with respect to any model parameter, we compute $\partial[X]/\partial\eta$ for $\eta = V$, U_k, or $S_{kk'}$ as follows:

$$\partial[X]/\partial\eta = \partial([X][A])/\partial\eta = (\partial[X]/\partial\eta)[A] + [X](\partial[A]/\partial\eta) \qquad [I5]$$

$$\rightarrow \qquad [I-A](\partial[X]/\partial\eta) = [X](\partial[A]/\partial\eta) \qquad [I6]$$

Due to the singularity of [I-A], this system requires an additional constraint; since $\sum_j \partial(x_j)/\partial\eta$ must be zero, we replace the last column of [I-A] by a column of ones, as before, giving us A*; next, replacing the last column of $(\partial[A]/\partial\eta)$ by a column of zeros, we rename this matrix D_η. Having done so, we have captured all constraints necessary to solve the system [I6]:

$$\partial[X]/\partial\eta = [X][D_\eta][A^*]^{-1} = e_\eta[A^*]^{-1}[D_\eta][A^*]^{-1} \qquad [I7]$$

We see that, to calculate $\partial[X]/\partial\eta$ for any model parameter of interest ($\eta = V$, U_k, or $S_{kk'}$), the only additional information needed beyond the original transition matrix A is the matrix $\partial[A]/\partial\eta$, which is easily transformed to D_η.

Solution by Successive Iteration

It is possible to obtain accurate approximations to the values of market shares and their derivatives through iteration. The approximations discussed in the n-brand analysis are the simplest examples. To solve for [X] or for $\partial[X]/\partial\eta$, we define the m^{th}-order iterations or m-iterates for these quantities ($[X_{(m)}]$

and $\partial[X_{(m)}]/\partial\pi)$ by the equations:

$$[X_{(m)}] = [X_{(m-1)}][A] \qquad [I8]$$

$$\partial[X_{(m)}]/\partial\pi) = \partial[X_{(m-1)}/\partial\pi)[A] + [X](\partial[A]/\partial\pi) \qquad [I9]$$

It is necessary, in addition, to specify the 0-iterates; since this is analogous to not 'running' the Markov process at all, we have $[X_{(0)}] = [\pi]$ (where $[\pi]$ denotes the 1xn matrix of brand preferences) and $\partial[X_{(m)}/\partial\pi = [0]$ (a 1xn matrix of zeros). Adopting this method, we use the 1-iterates for [X] and $\partial[X]/\partial\pi$ as approximations for the exact solutions as given by [I3] and [I7]:

$$[X_{(1)}] = [\pi][A] \quad \rightarrow \quad x_i \approx \sum_j a_{ji}\pi_j \qquad [I10]$$

$$\partial[X_{(1)}/\partial\pi) = [0][A] - [X](\partial[A]/\partial\pi) = [X](\partial[A]/\partial\pi) \qquad [I11]$$

Empirical evidence has proven these estimates to be quite good. Values given by [I8] and [I9] for the 50-iterates produce errors on the order of 10^{-5} when compared with the corresponding exact solutions ([I3] and [I7]; for 1-iterates, regressions of approximated values on exact values produce adjusted R^2's well above 90%, with constant terms insignificant. We therefore infer that 1-iterates provide excellent indicators of the direction and magnitude of exact values and are reliable in analyses at the individual or aggregate level.

A more direct justification for the use of 1-iterates derives from the assumption that $\partial[X]/\partial\pi$ is small for any choice of π. This says that, in

solving for [X], brand preference is a much stronger force on choice than variety-seeking, so the effect of successive iterations is small, and the Markov process converges quickly. Scrutinizing the functional form of equation [I5], assuming $\partial[X]/\partial\pi$ is small means replacing it by a row vector of zeros on the right side of the equation, precisely the assumption used to obtain equation [I11] for the 1-iterate derivatives.

Changing V

We have previously noted that all that is necessary for the computation of $\partial[X]/\partial\pi$ for any choice of π is the matrix $\partial[A]/\partial\pi$, in this case, $\partial[A]/\partial V$; we merely need the individual entries $\partial(a_{ji})/\partial V$:

$$\partial(a_{ji})/\partial V = \partial[(\pi_i - VS_{ij})/(1-VS_{\cdot j})]/\partial V$$

$$= (S_{\cdot j}\pi_i - S_{ij})/(1-VS_{\cdot j})^2 \qquad [I12]$$

As an approximation to the solution given by [I7] with $\pi=V$, we invoke the 1-iterate as given by [I11] and the values of $\partial(a_{ji})/\partial V$ from [I12]:

$$\partial[X_{(1)}]/\partial V) = [X](\partial[A]/\partial V) \qquad [I13]$$

$$\rightarrow \quad \partial x_i/\partial V \approx \sum_j x_j(S_{\cdot j}\pi_i - S_{ij})/(1-VS_{\cdot j})^2 \qquad [I14]$$

Changing π_1

We approach this derivative in a fashion entirely analogous to that of the preceding section, calculating only $\partial(a_{ji})/\partial\eta$. Since there is no preferred numbering scheme for the brands in a market, no generality is lost by computing $\partial[X]/\partial U_1$ (as opposed to $\partial[X]/\partial U_k$ for any k). Following the discussion of sharing rules for apportioning U_1 discussed in the n-brand case, we note that the addition of a unique feature of value ε to U_1 requires that each of the model's other feature parameters (ie. U's and S's) be rescaled so that the ratio of its old value to its new value is constant across all such parameters. Recalling that the sum of the scaled preferences is constrained, $\sum_j \pi_j = 1$ implies that, for any feature parameter η:

$$\partial\eta/\partial U_1 = \begin{cases} -\eta/(1-U_1) & \text{for } \eta \neq U_1 \\ 1 & \text{for } \eta = U_1 \end{cases} \qquad [I15]$$

This allows us to calculate the required matrix elements:

$$\partial a_{ji}/\partial U_1 = \begin{cases} -a_{ji}/(1-U_1)(1-VS_{.j}) & \text{for } i\neq 1,\ j\neq 1 \\ (1-a_{ji})/(1-U_1)(1-VS_{.j}) & \text{for } i=1,\ j\neq 1 \\ -a_{ji}(1-V)/(1-U_1)(1-VS_{.j}) & \text{for } i\neq 1,\ j=1 \\ (1-a_{ji})(1-V)/(1-U_1)(1-VS_{.j}) & \text{for } i=1,\ j=1 \end{cases} \qquad [I16]$$

giving us the associated 1-iterate approximation as well:

$$\partial x_i/\partial U_1 \approx \sum_{j=1}^{n} (\partial a_{ji}/\partial U_1)(x_j) \qquad [I17]$$

Changing S_{12} with π's Held Constant

Once again, without loss of generality, we choose two brands, B_1 and B_2, with which to carry out our analysis. Since only S_{12} is changing, none of the π_j change for any j, and of the U_j, only U_1 and U_2 change their value in that they decrease by the same amount by which S_{12} increases. Calculation of the matrix of derivatives, therefore, is straightforward, since a_{ji} is entirely independent of S_{ij} for i and j both greater than 2. Therefore:

$$\partial a_{ji}/\partial S_{12} = \begin{cases} 0 & \text{for } j>2 \\ Va_{ji}/(1-VS_{.j}) & \text{for } (j=1, i\neq 2) \text{ or } (j=2, i\neq 1) \\ V(a_{ji}-1)/(1-VS_{.j}) & \text{for } (j=1, i=2) \text{ or } (j=2, i=1) \end{cases} \quad [I18]$$

This yields the following 1-iterate approximations:

$$\partial x_1/\partial S_{12} \approx [Va_{11}/(1-VS_{.1})](x_1) - [V(a_{21}-1)/(1-VS_{.2})](x_2) \quad [I19]$$

$$\partial x_2/\partial S_{12} \approx [V(a_{12}-1)/(1-VS_{.1})](x_1) + [Va_{22}/(1-VS_{.2})](x_2)$$

$$\partial x_k/\partial S_{12} \approx [Va_{1k}/(1-VS_{.1})](x_1) + [Va_{2k}/(1-VS_{.2})](x_2) \quad \text{for } k>2$$

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